

# Covariant Quark-Representation of Composite Meson Systems and Chiral Symmetry

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(Received May 20, 2000)

Assuming the spin-independence for confining force, we give a covariant quark representation of general composite meson systems with definite Lorentz transformation properties. For benefit of this representation we are able to deduce automatically the transformation rules of composite mesons for general symmetry operations from those of constituent (exciton) quarks. Applying this we investigate especially physical implication of chiral symmetry for the meson systems, and point out a possibility of existence of new meson multiplets.

## §1. Introduction

There are the two contrasting view points of composite quark-antiquark mesons: The one is non-relativistic, based on the approximate symmetry of  $LS$ -coupling in the non-relativistic quark model (NRQM); while the other is relativistic, based on the dynamically broken chiral symmetry typically displayed in the Nambu Jona-Lasinio (NJL) model. The  $\pi$ -meson (or  $\pi$ -nonet) is now widely believed to have a dual nature of non-relativistic particle with  $(L, S) = (0, 0)$  and also of relativistic particle as a Nambu-Goldston boson with  $J^P = 0^-$  in the case of spontaneous breaking of chiral symmetry. However, no successful attempts to unify the above two view points have been yet proposed. On the other hand we have developed the covariant oscillator quark model (COQM)<sup>2) - 6)</sup> for many years as a covariant extension of NRQM, which is based on the boosted  $LS$ -coupling scheme. The meson wave functions (WF) in COQM are tensors in the  $\tilde{U}(4) \otimes O(3, 1)$  space and reduce at the rest frame to those in the  $SU(2)_{\text{spin}} \otimes O(3)_{\text{orbit}}$  space in NRQM. The COQM has been successful especially in treating the  $Q\bar{Q}$  meson system and the  $(q, \bar{Q})$  or  $(Q, \bar{q})$  meson system, leading, respectively, to a satisfactory understanding of radiative transitions and to the same weak form factor relations as in the heavy quark effective theory (HQET). However, in COQM no consideration on chiral symmetry has been given and it is not able to explain the dual nature of  $\pi$  meson.

The purpose of this paper is to get rid of this defect in COQM and is to give a unified view point of the two contrasting ones of the composite meson systems, extending the tensors of WF from the restricted ones necessary only in the boosted  $LS$  coupling scheme to the general ones in the  $\tilde{U}(4) \otimes O(3, 1)$  space, which are required for taking into account chiral symmetry.

## §2. Covariant Framework for Describing Composite Mesons

For meson WF described by  $\Phi_A^B(x_1, x_2)$  ( $x_1, x_2$  denoting the space-time coordinate and  $A = (\alpha, a)(B = (\beta, b))$  denoting the Dirac spinor and flavor indices of constituent quark (anti-quark)) we set up the bilocal Yukawa equation<sup>4)</sup>

$$\left[ \frac{\partial^2}{\partial X_\mu^2} - \mathcal{M}^2(x_\mu, \frac{\partial}{\partial x_\mu}) \right] \Phi_A^B(X, x) = 0 \quad (2.1)$$

( $X(x)$  denoting the center of mass (CM) (relative) coordinate of meson), where the  $\mathcal{M}^2$  is squared mass operator including only a central, Dirac-spinor-independent\*) confining potential. The WF is separated into the plane wave describing CM motion and the (Fierz-component) internal WF as

$$\Phi_A^B(x_1, x_2) = \sum_{\mathbf{P}_n, n} (e^{iP_n X} \Psi_{n,A}^{(+)\,B}(x, P_n) + e^{-iP_n X} \Psi_{n,A}^{(-)\,B}(x, P_n)), \quad (2.2)$$

where the Fierz components  $\Psi_n^{(\pm)}$  are eigenfunctions of  $\mathcal{M}^2$  as

$$\mathcal{M}^2(x_\mu, \frac{\partial}{\partial x_\mu}, P_n) \Psi_n^{(\pm)} = M_n^2 \Psi_n^{(\pm)}, \quad (2.3)$$

$P_{n,\mu}^2 = -M_n^2$ ,  $P_{n,0} = \sqrt{M_n^2 + \mathbf{P}_n^2}$ ; and the label  $(\pm)$  represents the positive (negative) frequency part; and  $n$  does a freedom of excitation. We have the following field theoretical expression for the WF in mind as a guide for developing the present semi-phenomenological approach:

$$\Phi_A^B(x_1, x_2) = \sum_n [\langle 0 | \psi_A(x_1) \bar{\psi}^B(x_2) | M_n \rangle + \langle M_n^c | \psi_A(x_1) \bar{\psi}^B(x_2) | 0 \rangle], \quad (2.4)$$

where  $\psi_A(\bar{\psi}^B)$  denotes the quark field (its Pauli-conjugate) and  $|M_n\rangle$  ( $\langle M_n^c|$ ) does the composite meson (its charge conjugate) state, and the first (second) term in the RHS corresponds to the positive (negative) frequency part in Eq. (2.2). The internal WF is, concerning the Dirac-spinor-dependence, expanded in terms of a complete set  $\{W_i\}$  of free bi-Dirac spinors of quarks and anti-quarks; and the internal WF is expressed as

$$\begin{aligned} \Psi_A^{(\pm)\,B}(x, P_n) &= \sum_i W_{i\alpha}^{(\pm)\beta}(P_n) \phi_a^{(\pm)b}(x, P_n), \\ \phi^{(\pm)}(x, P_n) &= \epsilon_i \langle \bar{W}_i^{(\mp)} \Psi^{(\pm)} \rangle, \end{aligned} \quad (2.5)$$

where  $\langle A \rangle$  means trace of  $A$ . The ortho-normal relations  $\langle \bar{W}_i^{(\mp)} W_j^{(\pm)} \rangle = \epsilon_i \delta_{ij}$  holds for the Pauli-conjugate of WF, defined by  $\bar{W}_i^{(\mp)} \equiv \gamma_4 W_i^{(\pm)\dagger} \gamma_4$ , where  $\bar{W}_i^{(\mp)}$  is related with the  $W_i^{(\mp)}$ , and the  $\epsilon_i$  and  $\delta_{ij}$  denote, respectively, the sign and the Kronecker symbols, see the appendix A).

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<sup>\*)</sup> In the boosted  $LS$ -coupling scheme the squared mass operator  $\mathcal{M}^2$  was assumed to be only Pauli-spinor independent, while in the present scheme it is assumed more generally to be Dirac-spinor independent.

### §3. Complete Set of Spin Wave Function and Composite Mesons with Definite Spin

We set up the conventional “free” Dirac spinors with four-momentum of composite meson itself  $P = P_M$ ,  $D_{q,\alpha}(P) \equiv (u_{q,\alpha}(P, s_q), v_{q,\alpha}(P, s_q))$  ( $s_q = \pm$  representing the spin up-down)) for quarks and  $\bar{D}_{\bar{q}}^\beta(P) \equiv (\bar{v}_{\bar{q}}^\beta(P, s_{\bar{q}}), \bar{u}_{\bar{q}}^\beta(P, s_{\bar{q}}))$  ( $s_{\bar{q}} = \pm$  representing spin up-down)) for anti-quarks. It is to be noted that all four spinors for both “quarks and anti-quarks” are necessary\*) to describe the spin WF of mesons. Then the complete set of bi-Dirac spinors is given by\*\*)

$$\{W^{(+)}(P)\} : \begin{aligned} U(P) &= u_q(p_1, s_q) \bar{v}_{\bar{q}}(p_2, s_{\bar{q}})|_{p_{i,\mu}=\kappa_i P_\mu} = u_+(\mathbf{P}, s_q) \bar{v}_+(\mathbf{P}, s_{\bar{q}}), \\ C(P) &= u_q(p_1, s_q) \bar{u}_{\bar{q}}(p_2, s_{\bar{q}})|_{p_{i,\mu}=\kappa_i P_\mu} = u_+(\mathbf{P}, s_q) \bar{v}_-(\mathbf{P}, s_{\bar{q}}), \\ D(P) &= v_q(p_1, s_q) \bar{v}_{\bar{q}}(p_2, s_{\bar{q}})|_{p_{i,\mu}=\kappa_i P_\mu} = u_-(\mathbf{P}, s_q) \bar{v}_+(\mathbf{P}, s_{\bar{q}}), \\ V(P) &= v_q(p_1, s_q) \bar{u}_{\bar{q}}(p_2, s_{\bar{q}})|_{p_{i,\mu}=\kappa_i P_\mu} = u_-(\mathbf{P}, s_q) \bar{v}_-(\mathbf{P}, s_{\bar{q}}), \end{aligned} \quad (3.1)$$

where  $u_+(\mathbf{P})(\bar{v}_+(\mathbf{P}))$  and  $u_-(\mathbf{P})(\bar{v}_-(\mathbf{P}))$  denote the Dirac spinors with positive energy and momentum  $\mathbf{P}$  and with negative energy and momentum  $-\mathbf{P}$ , respectively, describing quarks (anti-quarks). These energy and momentum concern with the total meson, while in Eq.(3.1) we have defined technically the momenta of “constituent quarks”\*\*\*) as

$$\begin{aligned} p_{i,\mu} &\equiv \kappa_i P_\mu, \quad p_{i,\mu}^2 = -m_i^2; \quad P_\mu^2 = -M^2, \quad M = m_1 + m_2 \\ (\kappa_{1,2} &\equiv m_{1,2}/(m_1 + m_2); \quad \kappa_1 + \kappa_2 = 1). \end{aligned} \quad (3.2)$$

The respective members in Eq.(3.1) satisfy a couple of the corresponding free Dirac equations in momentum space (which are equivalent to the (conventional or new-type of) Bargman-Wigner Equations) and are expressed in terms of their irreducible composite meson WF as follows:

(Non Rela. comp.)

$$\begin{aligned} (iP\gamma^{(1)} + M)U(P) &= 0, \quad U(P)(-iP\gamma^{(2)} + M) = 0; \\ U_A^B(P) &= \frac{1}{2\sqrt{2}}[(i\gamma_5 P_{s,a}^{(NR)b}(P) + i\gamma_\mu V_{\mu,a}^{(NR)b}(P))(1 + \frac{iP \cdot \gamma}{M})]_{\alpha}^{\beta}, \end{aligned}$$

(Semi Rela. comp.)

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\*) For understanding this it may be useful to take an analogy of the Bethe-Salpeter amplitude of deuteron. In expanding the amplitude all  $4 \times 4$  members of direct product of both Dirac spinors for constituent proton and neutron are necessary.

\*\*) In the following from §3 to §4.3 we give only the expressions of (+)-frequency parts, and consider only the ground states of composite system, disregarding the relative coordinates. In Eq.(3.1) considerations on the freedom of Pauli-spin are neglected. We have given detailed considerations on this problem and useful formulas in Appendix A.

\*\*\*) In so far as concerned with Eqs. (3.1) and (3.2) the quantities  $\kappa_i$  and accordingly  $m_i$  are arbitrary and have no physical meaning. However,  $m_i$  have proved to be the effective masses of constituent quarks through the phenomenological applications<sup>5), 6)</sup> of COQM so far made.

$$\begin{aligned}
\bar{q}\text{-type} \quad & (iP \cdot \gamma^{(1)} + M)C(P) = 0, \quad C(P)(iP \cdot \gamma^{(2)} + M) = 0; \\
& C_A^B(P) = \frac{1}{2\sqrt{2}}[(S_a^{(\bar{q})b}(P) + i\gamma_5 \gamma_\mu A_{\mu,a}^{(\bar{q})b}(P))(1 - \frac{iP \cdot \gamma}{M})]_\alpha^\beta, \\
q\text{-type} \quad & (-iP \gamma^{(1)} + M)D(P) = 0, \quad D(P)(-iP \gamma^{(2)} + M) = 0; \\
& D_A^B(P) = \frac{1}{2\sqrt{2}}[(S_a^{(q)b}(P) + i\gamma_5 \gamma_\mu A_{\mu,a}^{(q)b}(P))(1 + \frac{iP \cdot \gamma}{M})]_\alpha^\beta, \\
(\text{Extrly. Rel. comp.}) \quad & (-iP \gamma^{(1)} + M)V(P) = 0, \quad V(P)(iP \gamma^{(2)} + M) = 0; \\
& V_A^B(P) = \frac{1}{2\sqrt{2}}[(i\gamma_5 P_{s,a}^{(ER)b}(P) + i\gamma_\mu V_{\mu,a}^{(ER)b}(P))(1 - \frac{iP \gamma}{M})]_\alpha^\beta, \quad (3 \cdot 3)
\end{aligned}$$

where all vector and axial-vector mesons satisfy the Lorentz conditions,  $P_\mu V_\mu(P) = P_\mu A_\mu(P) = 0$ . Here it is to be noted that, in each type of the above members, the number of freedom counted both in the quark representation and in the meson representation is equal, as it should be ( $2 \times 2 = 4$  and  $1 + 3 = 4$ , respectively). Also it may be amusing to note that each constituent quarks in all the above members is in “parton-like motion,” having the same 3-dimensional velocity as that of total mesons. (For example, in  $V(P)$ ,  $\mathbf{v}_{1,2} = \frac{\mathbf{p}^{(1,2)}}{p_0^{(1,2)}} = \frac{-\kappa_{1,2} \mathbf{P}_M}{-\kappa_{1,2} P_{M,0}} = \mathbf{v}_M$ .)

#### §4. Transformation Properties of Composite Meson Systems and Chiral Symmetry

For benefit of covariant quark-representation of mesons given in §3 we can deduce automatically the transformation rules of composite mesons for general symmetry operations as follows:

##### 4.1. Charge conjugation

Charge conjugation properties of the bi-spinors and, correspondingly, of the composite mesons are derived from those of quarks as follows:

$$\begin{aligned}
\text{quark field :} \quad & \psi_\alpha(x) \rightarrow \psi_\alpha^c(x) = U_C^{-1} \psi_\alpha(x) U_C = (C^{-1})_{\alpha\alpha'} \bar{\psi}^{\alpha'}(x) \\
& \bar{\psi}^\beta(x) \rightarrow \bar{\psi}^{c\beta}(x) = U_C^{-1} \bar{\psi}^\beta(x) U_C = C^{\beta\beta'} \psi_{\beta'}(x) \quad (4 \cdot 1) \\
& (CC^\dagger = 1, \quad C = \gamma_4 \gamma_2, \quad C \gamma_\mu C^{-1} = -{}^t \gamma_\mu).
\end{aligned}$$

Internal meson WF :

$$\begin{aligned}
\Psi_A^{(+B)}(P, x) & \approx \langle 0 | \psi_A(x_1) \bar{\psi}^B(x_2) | M \rangle \\
& \rightarrow \Psi_A^{c, (+B)}(P, x) (\approx \langle 0 | \psi_A(x_1) \bar{\psi}^B(x_2) | M^c \rangle) \\
& = \langle 0 | U_C^{-1} \psi_A(x_1) U_C U_C^{-1} \bar{\psi}^B(x_2) U_C | M \rangle \\
& = \langle 0 | (C^{-1})_{AA'} \bar{\psi}^{A'}(x_1) C^{BB'} \psi_{B'}(x_2) | M \rangle \\
& = (C^{-1})_{AA'} (-1) \langle 0 | \psi_{B'}(x_2) \bar{\psi}^{A'}(x_1) | M \rangle ({}^t C)^{B'B} \\
& = (C^{-1})_{AA'} {}^t \Psi^{(+)}(P, -x)^{A'}_{B'} C^{B'B},
\end{aligned}$$

Spinor WF :

$$\begin{aligned}
U_{P_s}^{(NR)} &\leftrightarrow U_{P_s}^{(NR)}, & U_{V_\mu}^{(NR)} &\leftrightarrow -U_{V_\mu}^{(NR)}, \\
C_S^{\bar{q}} &\leftrightarrow D_S^q, & C_{A_\mu}^{\bar{q}} &\leftrightarrow D_{A_\mu}^q, \\
U_{P_s}^{(ER)} &\leftrightarrow U_{P_s}^{(ER)}, & U_{V_\mu}^{(ER)} &\leftrightarrow -U_{V_\mu}^{(ER)}, \\
\text{Composite meson WF :} \\
P_{s,a}^{(NR)b} &\leftrightarrow P_{s,b}^{(NR)a}, & V_{\mu,a}^{(NR)b} &\leftrightarrow -V_{\mu,b}^{(NR)a}, \\
S_a^{(\bar{q})b} &\leftrightarrow S_b^{(q)a}, & A_{\mu,a}^{(\bar{q})b} &\leftrightarrow A_{\mu,b}^{(q)a}, \\
P_{s,a}^{(ER)b} &\leftrightarrow P_{s,b}^{(ER)a}, & V_{\mu,a}^{(ER)b} &\leftrightarrow -V_{\mu,b}^{(ER)a}.
\end{aligned} \tag{4.2}$$

#### 4.2. Chiral transformation

Chiral transformation properties of composite mesons are also derived straightforwardly from those of the bi-spinors. For example, for  $SU(3)$  chiral transformation

$$\Psi_A^B(P, x) \rightarrow [e^{i\alpha^i \frac{\lambda^i}{2} \gamma_5} \Psi(P, x) e^{i\alpha^i \frac{\lambda^i}{2} \gamma_5}]_A^B. \tag{4.3}$$

leading to the results:

$$\begin{aligned}
[M^{(J)}(P)]'_i &= [{}^t S^{(J)}(P, \alpha)]_{ij} [M^{(J)}(P)]_j \quad J = (0, 1) \\
M_i^{(0)} &\equiv {}^t [P_s^{(NR)}, S^{(R\bar{q})}, S^{(Rq)}, P_s^{(ER)}]_i, \\
M_i^{(1)} &\equiv {}^t [V_\mu^{(NR)}, A_\mu^{(R\bar{q})}, A_\mu^{(Rq)}, V_\mu^{(ER)}]_i,
\end{aligned} \tag{4.4}$$

where  $[S^{(J)}]$  denotes a unitary matrix, of which explicit form is omitted here. (For infinitesimal transformation, see appendix B).

#### 4.3. Light-quark meson system—“chiral $SU(6)$ multiplet”

The quark representation applying to the light-quark mesons is obtained by the linear transformation of the bi-Dirac spinors given in §3 as follows:

$$\begin{aligned}
U_{P_s, \alpha}^{(N,E)\beta} &\equiv \frac{1}{\sqrt{2}} (U_{P_s} \pm V_{P_s})_\alpha^\beta = \frac{1}{2} [(i\gamma_5, -\gamma_5 v \cdot \gamma)]_\alpha^\beta; \quad P_s^{(N,E)}; C = (+, +) \\
C_{S, \alpha}^{(N,E)\beta} &\equiv \frac{(1, i)}{\sqrt{2}} (D_S \pm C_S)_\alpha^\beta = \frac{1}{2} [(1, -v \cdot \gamma)]_\alpha^\beta; \quad S^{(N,E)}; C = (+, -) \\
U_{V, \alpha}^{(N,E)\beta} &\equiv \frac{1}{\sqrt{2}} (U_V \pm V_V)_\alpha^\beta = \frac{1}{2} [(i\tilde{\gamma}_\mu, -i\sigma_{\mu\nu} v_\nu)]_\alpha^\beta; \quad V^{(N,E)}; C = (-, -) \\
C_{A, \alpha}^{(N,E)\beta} &\equiv \frac{(1, i)}{\sqrt{2}} (D_A \pm C_A)_\alpha^\beta = \frac{1}{2} [(i\gamma_5 \tilde{\gamma}_\mu, \gamma_5 \sigma_{\mu\nu} v_\nu)]_\alpha^\beta; \quad A^{(N,E)}; C = (+, -)
\end{aligned} \tag{4.5}$$

( $v_\mu \equiv P_\mu/M$ ,  $\tilde{\gamma}_\mu v_\mu \equiv 0$ ; and  $U_{P_s}$  denotes the coefficient bi-spinors of  $P_s$  and so on), where we have given also the charge-conjugation parity of the corresponding (hidden flavor) composite mesons. The chiral transformation properties of the new bi-spinors are easily seen to be similar as the conventional ones as

$$\begin{aligned}
1 &\leftrightarrow i\gamma_5, \quad -v \cdot \gamma \leftrightarrow -\gamma_5 v \cdot \gamma, \\
i\gamma_5 \tilde{\gamma}_\mu &\leftrightarrow i\tilde{\gamma}_\mu, \quad -i\sigma_{\mu\nu} v_\nu \leftrightarrow \gamma_5 \sigma_{\mu\nu} v_\nu.
\end{aligned} \tag{4.6}$$

#### 4.4. “Local chiral $SU(6)$ field”

Extending our considerations to include the  $(-)$ -frequency part, we are led to a unified expression of what to be called, Local Chiral  $SU(6)$  field, as

$$\begin{aligned}
\Psi_A^B(X) &= \Psi_A^{(N)B}(X) + \Psi_A^{(E)B}(X) \\
\Psi_A^{(N)B} &= \frac{1}{2} \left[ i\gamma_5 \alpha^\beta P_{s,a}^{(N)b} + i\tilde{\gamma}_{\mu,\alpha}^\beta V_{\mu,a}^{(N)b} + 1_\alpha^\beta S_a^{(N)b} + (i\gamma_5 \tilde{\gamma}_\mu)_\alpha^\beta A_{\mu,a}^{(N)b} \right] \\
\Psi_A^{(E)B} &= \frac{1}{2} \left[ (i\gamma_5 \gamma_\mu)_\alpha^\beta \frac{\partial_\mu}{\sqrt{\partial^2}} P_{s,a}^{(E)b} + (\sigma_{\mu\nu})_\alpha^\beta \frac{\partial_\mu}{\sqrt{\partial^2}} V_{\nu,a}^{(E)b} \right. \\
&\quad \left. + i\gamma_{\mu,\alpha}^\beta \frac{\partial_\mu}{\sqrt{\partial^2}} S_a^{(E)b} + (i\gamma_5 \sigma_{\mu\nu})_\alpha^\beta \frac{\partial_\mu}{\sqrt{\partial^2}} A_{\nu,a}^{(E)b} \right]. \tag{4.7}
\end{aligned}$$

### §5. Covariant Classification and Spectroscopy of Mesons and Chiral Symmetry

#### 5.1. Level classification of ground states

In the previous sections §2 and §3 we have presented a general covariant kinematical framework for describing the (ground states of) composite meson systems with a definite total quark spin. However, what kinds of mesons do really exist or not, that is, the meson spectroscopy, is a dynamical (still unsolved) problem of QCD.

For this problem, it is useful to apply in our scheme a physical consideration of dynamically broken chiral symmetry of QCD, typically displayed in the NJL model: In the (ground state of) heavy quarkonium ( $Q\bar{Q}$ ) system both quarks( $Q$ ) and antiquarks( $\bar{Q}$ ) are possible to do, since  $m_Q > \Lambda_{\text{conf}}$ , only non-relativistic motions with positive energy, and the non-relativistic  $LS$ -symmetry is good. Accordingly the bi-spinor  $U$  is considered to be applied to  $Q\bar{Q}$  system as a covariant spin WF. In the (ground state of) heavy-light quark meson  $Q\bar{q}(q\bar{Q})$  system the anti-quarks(quarks) make, since  $m_q \ll \Lambda_{\text{conf}}$ , relativistic motions both with positive and negative energies, and the relativistic chiral symmetry concerning light antiquarks (quarks) is good. Accordingly both the bi-spinors  $U$  and  $C$  ( $U$  and  $D$ ) are to be applied to the  $Q\bar{q}(q\bar{Q})$  system, and in this system there is a possibility of existence of new composite scalar and axial-vector mesons(see Eq.(3.3)). In the (ground state of) light quark  $q\bar{q}$ -meson system both quarks  $q$  and anti-quarks  $\bar{q}$  make, since  $m_q \ll \Lambda_{\text{conf}}$ , relativistic motions with both positive and negative energies, and chiral symmetry is good. Accordingly the linear combinations (specified in §4.3) of bispinors  $U$  and  $V$  are applied to the  $q\bar{q}$ -system, and in this system there is a possibility of existence of an extra(, in addition, to a normal) set of composite pseudo-scalar and vector mesons. Furthermore, the linear combinations of  $C$  and  $D$  are also applied, and normal and extra sets of composite scalar and axial-vector mesons possibly exist as relativistic  $S$ -wave bound states.

In the above discussion on the validity of chiral symmetry, we have supposed that  $\Lambda_{\text{conf}} \sim 1\text{GeV}$  regardless of quark-flavor. Here it may be useful to note that the positive (negative) energy Dirac spinors with momentum  $\mathbf{P}$  for quarks and an-

Table I. Level structure of the ground states of general quark meson systems

[Heavy quark system]		Mass	Spin WF	Type of Mesons
$m_q < \Lambda_{\text{conf}}$ : $q$ Rela. ( $q$ chiral sym. good)	$q\bar{Q}$	$m_q + m_{\bar{Q}}$	$u_+\bar{v}_+; u_-\bar{v}_+$	$P_s, V_\mu; S, A_\mu$
$m_{\bar{q}} < \Lambda_{\text{conf}}$ : $\bar{q}$ Rela. ( $\bar{q}$ chiral sym. good)	$Q\bar{q}$	$m_Q + m_{\bar{q}}$	$u_+\bar{v}_+; u_+\bar{v}_-$	$P_s, V_\mu; S, A_\mu$
$m_Q, m_{\bar{Q}} > \Lambda_{\text{conf}}$ : Non-R ( $LS$ sym. good)	$Q\bar{Q}$	$m_Q + m_{\bar{Q}}$	$u_+\bar{v}_+$	$P_s, V_\mu$
[Light quark system]				
$m_q, m_{\bar{q}} < \Lambda_{\text{conf}}$ : Rela. (chiral sym. good)	$q\bar{q}$	$m_q + m_{\bar{q}}$	$\frac{1}{\sqrt{2}}(u_+\bar{v}_+ \pm u_-\bar{v}_-)$ $\frac{(1-i)}{\sqrt{2}}(u_+\bar{v}_- \pm u_-\bar{v}_+)$	$P_s^{(N)}, V_\mu^{(N)}; P_s^{(E)}, V_\mu^{(E)}$ $S^{(N)}, A_\mu^{(N)}; S^{(E)}, A_\mu^{(E)}$

tiquarks change, respectively, into negative (positive) energy ones with momentum  $-\mathbf{P}$  under the operation of  $\gamma_5$  as

$$\gamma_5 u_+(\mathbf{P}) = u_-(-\mathbf{P}), \quad \bar{v}_+(\mathbf{P})\gamma_5 = \bar{v}_-(-\mathbf{P}). \quad (5.1)$$

The above expected level structure of the ground states of general light-through-heavy quark meson systems is summarized in Table I.

### 5.2. Level classification of excited states

In the last subsection we have stated on our level classification scheme, focusing on the ground-state mesons. In classifying the excited-state mesons we can proceed essentially similarly<sup>2)</sup> as the case of ground state mesons. In (the present extended version of) COQM in the pure-confining force limit, the masses of  $N$ -th excited states are given by the formula  $M_N^2 = M_G^2 + N\Omega$  ( $M_G \equiv M_0$ ,  $\Omega$  being the inverse Regge slope), and their covariant spin wave functions are defined by the same formulas as given in §3 with substitution of constituent exciton-quark mass  $m_i$  by

$$m_i^* = \gamma_N m_i \quad (\gamma_N \equiv M_N/M_G). \quad (5.2)$$

The value of confinement momentum  $\Lambda_{\text{conf}}$  is conventionally considered to be

$$\Lambda_{\text{conf}} \sim 1\text{GeV}. \quad (5.3)$$

The value of  $m_i^*$  for respective quark-configuration meson systems obtained by the formula (5.2) are given in Table II. From this table we see that  $m^* \ll \Lambda_{\text{conf}}$  for the lower levels, especially for the ground and first-excited states in the light-quark meson systems, and accordingly we can expect that chiral symmetry for these states may be still good. Similarly, in the light/heavy quark meson systems, the chiral symmetry concerning the light quark is also good for the several lower levels.

The quantum numbers  $P$  and  $C$  are given for the respective orbitally  $L$ -th excited light-quark mesons as follows:

$$\begin{aligned}
P_s^{(N,E)} \otimes \{L\} \quad P &= (-1)^{L+1}, \quad C = (-1)^L \\
V_\mu^{(N,E)} \otimes \{L\} \quad P &= (-1)^{L+1}, \quad C = (-1)^{L+1} \\
S^{(N,E)} \otimes \{L\} \quad P &= (-1)^L, \quad C = \pm(-1)^L \\
A_\mu^{(N,E)} \otimes \{L\} \quad P &= (-1)^L, \quad C = \pm(-1)^L.
\end{aligned} \quad (5.4)$$

Table II. The values of exciton-quark mass  $m_i^*$  for  $q\bar{q}$  and  $q\bar{Q}$  meson systems. In the case of  $m_i^* < \Lambda_{\text{conf}} \simeq 1$  GeV we can expect that the chiral symmetry for the corresponding meson system is still effective. The values of parameter  $\beta$ , representing the inverse of space-time extension of the system, and of the oscillator quanta  $\Omega$  are also given for reference. The  $\Omega$  is related with  $\beta$  by  $\Omega = (2(m_1 + m_2)^2 / (m_1 m_2))\beta$ . The values<sup>5)</sup> of constituent quark masses are taken as  $m_n = 0.375$ ,  $m_s = 0.51$ ,  $m_c = 1.70$  and  $m_b = 5.00$  (in GeV).

	$n\bar{n}$	$n\bar{s}$	$s\bar{s}$	$n\bar{c}$	$s\bar{c}$	$n\bar{b}$	$s\bar{b}$
$\Omega/\text{GeV}^2$	1.09	1.14	1.13	1.96	1.70	4.56	3.71
$\beta/\text{GeV}^2$	0.137	0.139	0.142	0.145	0.151	0.148	0.156
$(m_{q_1}^*, m_{q_2}^*)$	$m_n^*$	$(m_n^*, m_s^*)$	$m_s^*$	$(m_n^*, m_c^*)$	$(m_s^*, m_c^*)$	$(m_n^*, m_b^*)$	$(m_s^*, m_b^*)$
$(L=0)$	0.375	(0.375, 0.51)	0.51	(0.375, 1.7)	(0.51, 1.7)	(0.375, 5)	(0.51, 5)
$(L=1)$	0.64	(0.59, 0.80)	0.74	(0.45, 2.1)	(0.59, 2.0)	(0.40, 5.4)	(0.54, 5.3)
$(L=2)$	0.83	(0.74, 1.0)	0.9	(0.52, 2.3)	(0.66, 2.2)	(0.43, 5.7)	(0.57, 5.6)

### 5.3. Expected spectroscopy of mesons

In our fundamental equations Eqs.(2.1) to (2.3) it was supposed that the squared-mass spectra  $\mathcal{M}^2$  is, as a first step in the pure-confining force limit, Dirac-spinor independent and also quark-flavor independent.\*) Actually we must take into account<sup>5)</sup> the various effects due to one-gluon-exchange potential\*\*) and also the effects due to quark mass difference.

light – quark ( $q\bar{q}$ ) meson system: In the pure-confining force limit all the ground state mesons expected in §3.1,  $P_s^{(N,E)}$ ,  $V_\mu^{(N,E)}$ ,  $S^{(N,E)}$  and  $A_\mu^{(N,E)}$  are degenerate and have the same mass,  $M_0 = m_1 + m_2$ . Actually the mass of  $P_s^{(N)}$ , to be assigned  $\pi$ -nonet, should be exceptionally low because of its nature\*\*\*) as a Nambu-Goldstone boson. The masses of the  $V_\mu^{(N)}$ -nonet, which is assigned to be  $\rho$ -meson nonet, are almost equal to the corresponding masses  $M_0 = m_1 + m_2$  in the pure-confining force limit. The masses of all the other ground-state mesons are expected to be almost equal to those of the corresponding normal vector mesons, and to be lower than those of the corresponding first-excited states.

For the first-excited states, the chiral symmetry is expected to be still effective, as is seen from Table II given in the last subsection §5.2, so we expect the existence of a series of the first excited  $P$ -wave states of the ground state multiplets. They are expected to have the masses, which are almost equal to the first excited states of normal vector mesons, and which are lower than the second excited states of those. Among the multiplets newly predicted in the present scheme, to be called “chiralons,” the especially interesting mesons are the ones with  $J^{PC}=0^{+-}(S^{(E)}(S\text{-wave}))$ ,  $1^{-+}(S^{(E)}(P\text{-wave}))$  and  $1^{-+}(A_\mu^{(E)}(P\text{-wave}))$ , which are “exotic particles” out of the conventional non-relativistic  $q\bar{q}$ -mesons. Their

\*) Of course we consider, separately, the light-quark system, light/heavy-quark system and heavy-quark system.

\*\*) We must also take the other non-perturbative QCD effects like quark-condensation and instantons.

\*\*\*) The  $q\bar{q}$ -condensation corresponds to the non-zero vacuum expectation value of  $S^{(N)}$ ,  $\langle S^{(N)} \rangle_0$ . Thus  $P_s^{(N)}$  other than  $P_s^{(E)}$  is a Nambu-Goldstone boson.



masses, by the above mentioned estimate, is expected to be, respectively,

$$\begin{aligned} m(0^{+-}) &\lesssim 1.3 \text{ GeV}, \\ 1.3 \text{ GeV} &\simeq m(1^{-+}, S^{(E)}) \simeq m(1^{-+}, A_{\mu}^{(E)}) \lesssim 1.7 \text{ GeV} \end{aligned} \quad (5.5)$$

heavy – light quark ( $Q\bar{q}$  and  $q\bar{Q}$ ) meson system: As is seen from Table I we are able to expect the existence of new multiplets (at least the ground states of scalar and axial-vector triplet).

heavy quark ( $Q\bar{Q}$ ) meson system: No new multiplets are expected to exist.

## §6. Experimental Evidences and Concluding Remarks

In this paper we have presented a kinematical framework for describing covariantly the ground states as well as excited states of light-through-heavy quark mesons. For light-quark mesons our scheme gives a theoretical basis to classify the composite meson systems unifying the two contrasting viewpoints based on non-relativistic quark model with  $LS$  symmetry and on NJL model with chiral symmetry. The essential physical assumption is to set up the Klein-Gordon type of Yukawa equation on the bi-local meson wave function with the squared-mass operator, which is, in the pure-confining force limit, independent of Dirac-spinor suffix, and accordingly is chiral symmetric. As a result is pointed out a possibility of existence of rather an abundant new nonets, chiralons, with masses lower than about 2 GeV; several new ground state meson nonets and some new excited meson nonets.

For heavy/light quark meson systems we have similarly pointed out a possibility of existence of new multiplets(triplets), chiralons.

In Table III we have summarized the expected level structure of general ground and the first-excited quark-antiquark mesons. Presently we can give a few experimental candidates for the predicted members of new multiplets: One of the most important and interesting ones is the scalar  $\sigma$  nonet; the members<sup>\*)</sup> are  $\sigma(600)$ ,  $\kappa(900)$ ,  $a_0(980)$  and  $f_0(980)$ , which constitute, with the members of  $\pi$ -nonet, a linear representation of the chiral  $SU(3)$  symmetry. It is notable that the  $\sigma$  nonet is the relativistic  $S$ -wave states, which should be discriminated from the non-relativistic  $^3P_0$  states.

Another example suggesting possible validity of the present scheme is existence of the three pseudoscalars with mass between 1 GeV~1.5GeV,  $\eta(1295)$ ,<sup>22), 23)</sup>  $\eta(1420)$ <sup>24), 22), 25)</sup> and  $\eta(1460)$ .<sup>25)</sup> The two out of them may be the members of the radially excited  $\pi$ -nonet, while the one extra may belong to the ground states of the extra pseudoscalar nonet newly predicted.

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<sup>\*)</sup> This assignment of  $\sigma$  nonet was first proposed in Ref. 7) and afterwards in Refs. 8), 9), and by one of the present authors in Ref. 10). Rather strong experimental evidences for  $\sigma(600)$  have been given recently by many authors; from the  $\pi\pi$  scattering process Refs. 11)-15) . and from the  $\pi\pi$  production processes Ref. 16)-19). Possible evidences for  $\kappa(900)$  was pointed out by reanalyzing the  $K\pi$  scattering phase shift in Refs. 8), 20) and 21).

Table III. Quantum numbers of Ground and Excited states of light through heavy quark-antiquark mesons and their experimental candidates. For  $Q\bar{q}$ ,  $q\bar{Q}$ -system, angular momentum of the “light degrees of freedom”  $j_q$ , which are obtained by  $\mathbf{j}_q = \mathbf{L} + \mathbf{S}_q$ , is expected to be good quantum number, and the two  $P$ -wave meson states with  $J = 1$ ,  $j_q = 1/2$  and  $3/2$ ,  $M(P_1^{j_q=1/2, 3/2})$  are the linear combinations of the conventional  $LS$ -states,  $M(^{1,3}P_1)$ , as  $M(P_1^{j_q=1/2}) = \sqrt{2/3}M(^3P_1) - \sqrt{1/3}M(^1P_1)$ , and  $M(P_1^{j_q=3/2}) = -\sqrt{1/3}M(^3P_1) - \sqrt{2/3}M(^1P_1)$ .

$q\bar{q}$	$J^{PC}$	Experimental candidates
$P_s^{(N)} \otimes \{L\}$ $V_\mu^{(N)} \otimes \{L\}$	$^1S_0$ $0^{-+}$ $^3S_1$ $1^{--}$ $^1P_1$ $1^{+-}$ $^3P_0$ $0^{++}$ $^3P_1$ $1^{++}$ $^3P_2$ $2^{++}$	$\pi, K, \eta, \eta'$ $\rho, K^*, \omega, \phi$ $h_1(1170), b_1(1230)$ $f_0(1370), K_0^*(1430), a_0(1450), (f_0(1500))$ $a_1(1230), f_1(1285), K_1(1270)$ $a_2(1320), f_2(1275), K_2^*(1430), f_2'(1525)$
$P_s^{(E)} \otimes \{L\}$ $V_\mu^{(E)} \otimes \{L\}$	$^1S_0$ $0^{-+}$ $^3S_1$ $1^{--}$ $^1P_1$ $1^{+-}$ $^3P_0$ $0^{++}$ $^3P_1$ $1^{++}$ $^3P_2$ $2^{++}$	(one out of $\eta(1275), \eta(1420)$ and $\eta(1460)$ )
$S^{(N)} \otimes \{L\}$ $A_\mu^{(N)} \otimes \{L\}$	$^1S_0$ $0^{++}$ $^1P_1$ $1^{--}$ $^3S_1$ $1^{++}$ $^3P_0$ $0^{--}$ $^3P_1$ $1^{--}$ $^3P_2$ $2^{--}$	$\sigma, \kappa, a_0(980)=\delta, f_0(980)=\sigma'$ $a_1(900)$
$S^{(E)} \otimes \{L\}$ $A_\mu^{(E)} \otimes \{L\}$	$^1S_0$ $\underline{0^{+-}}$ $^1P_1$ $\underline{1^{-+}}$ $^3S_1$ $1^{+-}$ $^3P_0$ $0^{-+}$ $^3P_1$ $\underline{1^{-+}}$ $^3P_2$ $\underline{2^{-+}}$	$\pi_1(1400)$ $\pi_1(1600)$
$Q\bar{q}, q\bar{Q}$	$J^P$	
$P_s \otimes \{L\}$ $V_\mu \otimes \{L\}$	$^1S_0$ $0^-$ $^3S_1$ $1^-$ $P_1^{j_q=1/2}$ $1^+$ $P_1^{j_q=3/2}$ $1^+$ $^3P_0$ $0^+$ $^3P_2$ $2^+$	$D, B, D_s, B_s$ $D^*, B^*, D_s^*, B_s^*$ $D_1^*$ $D_1$ $D_0^*$ $D_2^*$
$S \otimes \{L\}$ $A_\mu \otimes \{L\}$	$^1S_0$ $0^+$ $^3S_1$ $1^+$ $P_1^{j_q=1/2}$ $1^-$ $P_1^{j_q=3/2}$ $1^-$ $^3P_0$ $0^-$ $^3P_2$ $2^-$	$B_0^c(5520)$
$Q\bar{Q}$	$J^P$	
$P_s \otimes \{L\}$ $V_\mu \otimes \{L\}$	$^1S_0$ $0^-$ $^3S_1$ $1^-$ $^1P_1$ $1^+$ $^3P_0$ $0^+$ $^3P_1$ $1^+$ $^3P_2$ $2^+$	$\eta_c, B_c$ $J/\psi, \Upsilon, B_c^*$ $h_c$ $\chi_{c0}, \chi_{b0}$ $\chi_{c1}, \chi_{b1}$ $\chi_{c2}, \chi_{b2}$

Also we have the other candidates for chiralons: It was a problem for experimental groups for long time whether a possible resonance observed in the  $\eta\pi$  system, with an exotic quantum number  $J^{PC} = 1^{-+}$  and with a mass around 1.5 GeV, really exists or does not. Recently it seems that the existence of two such particles<sup>26)</sup>  $\pi_1(1400)$  and  $\pi_1(1600)$  have been accepted<sup>27)</sup> widely. These two particles have the mass in the region estimated in Eq.(5.5) to be assigned as the respective excited  $P$ -wave states of  $S^{(E)}$  and  $A_\mu^{(E)}$ .

We have the other longstanding problem in hadron spectroscopy: the mass and width of  $a_1(1260)$  seem to be variant<sup>28)</sup> depending<sup>29)</sup> on the production process and/or decay channel.

In connection to this problem we have made recently a preliminary analysis<sup>30)</sup> of the data<sup>31)</sup> obtained by GAMS group WA102 experiment on process  $\pi^-p \rightarrow 3\pi^0n$ . As a result we have obtained an evidence of existence of two  $a_1(J^{PC} = 1^{++})$  particles:  $a_1^c(m = 0.9\text{GeV}, \Gamma = 200\text{MeV})$ , and  $a_1^N(m = 1.2\text{GeV}, \Gamma = 440\text{MeV})$ . The former may be assigned to be  $A_\mu^{(N)}(^3S_1)$ , while the latter be conventional  $a_1$  particle (to be  $V_\mu^{(N)}(^3P_1)$  in our classification scheme).

All the above candidates for chiralons are concerned with the light-quark mesons. Here, we should like to refer to a preliminary result concerning the heavy/light quark meson systems that a scalar chiralons  $B_0^c$  with  $M=5.52$  GeV and  $\Gamma=44$  MeV may be observed<sup>32)</sup> in the  $B\pi$  channel produced<sup>33)</sup> through the  $Z$ -boson decay.

We have also given the above mentioned experimental candidates for chiralons in Table III, where we have listed, for reference, also our assignment to the conventional  $S$ -wave and  $P$ -wave states.

In concluding we should like to remark that in this paper we have dared to present a very "brave" attempt for unified classification scheme of mesons, which predicts the possible existence of a lot of new meson multiplets. Further serious investigations and search for them will be required to test validity of the present scheme.

### Acknowledgements

The authors would like to express their sincere gratitude to prof. K. Takamatsu, T. Tsuru and T. Sawada for encouragements and useful informations. They would like to thank prof. K. Yamada for useful comments. They are also grateful to Dr. T. Ishida for encouragements and comments.

### Appendix A

—— *Spin wave functions of composite meson systems*  
and *fundamental crossing rules for constituent quarks* ——

In the free local field its annihilation (positive frequency) and creation (negative frequency) parts are related with each other by the conventional crossing rule. In this appendix we shall derive a similar crossing rule for our covariant spin WF of composite meson systems by applying the fundamental crossing rule<sup>34)</sup> of our

extended Dirac spinors for constituent quarks. In the following we first make the annihilation part of the composite meson WF by decomposition of total spin of the constituent quarks and antiquarks. Next by using the fundamental crossing rules for the constituent spinors we construct the creation part of the composite meson spinor WF.

We use the following conventional “free” Dirac  $u$  and  $v$  spinors:

$$u(\mathbf{p}, h) = \begin{pmatrix} \sqrt{E+m}\chi^{(h)} \\ \sqrt{E-m}\mathbf{n}\cdot\boldsymbol{\sigma}\chi^{(h)} \end{pmatrix}, \quad v(\mathbf{p}, h) = \begin{pmatrix} \sqrt{E-m}\mathbf{n}\cdot\boldsymbol{\sigma}\chi^{(h)'} \\ \sqrt{E+m}\chi^{(h)'} \end{pmatrix} \quad (\text{A}\cdot 1)$$

In this Appendix A we choose the  $z$ -axis parallel to the momentum of composite mesons as  $\mathbf{p} = p\mathbf{n} = p\hat{\mathbf{z}}$ , and accordingly

$$\chi^{(+)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi^{(-)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \chi^{(h)'} \equiv -i\sigma_2\chi^{(h)*}, \quad \begin{matrix} \chi^{(+)' } & = & \chi^{(-)} \\ \chi^{(-)' } & = & -\chi^{(+)} \end{matrix} \quad (\text{A}\cdot 2)$$

#### A.1. quark and antiquark spinor inside of mesons

Here we give the annihilation part of the constituent quark and antiquark spinor WF. As was explained in the text the constituent quark inside of mesons has the freedom of positive and negative energy, as well as that of up and down spin, and totally four degrees of freedom.

The positive energy quark spinor  $u_+$  with the meson momentum  $\mathbf{p}$ , which is related with the “free” Dirac  $u$  spinor, is defined by

$$u_+(\mathbf{p}, \pm) = \begin{pmatrix} \sqrt{E+m}\chi^{(\pm)} \\ \sqrt{E-m}\mathbf{n}\cdot\boldsymbol{\sigma}\chi^{(\pm)} \end{pmatrix} = u(\mathbf{p}, \pm). \quad (\text{A}\cdot 3)$$

The negative energy quark spinor  $u_-$  with the momentum,  $-\mathbf{p}$ , anti-parallel to the meson momentum  $\mathbf{p}$ , is defined by using the same spin operator  $\mathbf{n}\cdot\boldsymbol{\sigma}$  as for  $u_+$ . They are related with free “Dirac”  $v$  spinors.

$$u_-(-\mathbf{p}, \pm) = \begin{pmatrix} \sqrt{E-m}\mathbf{n}\cdot\boldsymbol{\sigma}\chi^{(\pm)} \\ \sqrt{E+m}\chi^{(\pm)} \end{pmatrix} = \mp v(\mathbf{p}, \mp). \quad (\text{A}\cdot 4)$$

The positive energy antiquark spinor  $\bar{v}_+$  with momentum  $\mathbf{p}$ , which is related with the “free” Dirac  $\bar{v}$  spinor, is defined by

$$\bar{v}_+(\mathbf{p}, \pm) = (\sqrt{E-m}\chi^{(\pm)\dagger}\mathbf{n}\cdot\boldsymbol{\sigma}, -\sqrt{E+m}\chi^{(\pm)\dagger}) = \bar{v}(\mathbf{p}, \pm). \quad (\text{A}\cdot 5)$$

The negative energy antiquark spinor  $\bar{v}_-$  with momentum  $-\mathbf{p}$ , is defined, similarly by using the same spin operator  $-\mathbf{n}\cdot\boldsymbol{\sigma}$ , as in the case of  $\bar{v}_+$ . They are related with free “Dirac”  $\bar{u}$  spinors.

$$\bar{v}_-(-\mathbf{p}, \pm) = (\sqrt{E+m}\chi^{(\pm)\dagger}, -\sqrt{E-m}\chi^{(\pm)\dagger}\mathbf{n}\cdot\boldsymbol{\sigma}) = \pm \bar{u}(\mathbf{p}, \mp). \quad (\text{A}\cdot 6)$$

#### A.2. Fundamental crossing rules for constituent spinor inside of mesons

Under the crossing operation for mesons the annihilation (positive frequency) part of meson WF is transformed into the creation (negative frequency) part of that.

The corresponding fundamental crossing rule is set up as follows: For example, the positive energy constituent quark spinor in the annihilation part of meson WF is transformed into the positive energy antiquark spinor in the the creation part of meson WF.

$$u_+(\mathbf{p}, s) \longrightarrow v_+(\mathbf{p}, s) \quad \text{that is} \quad u(\mathbf{p}, \pm) \longrightarrow v(\mathbf{p}, \pm). \quad (\text{A}\cdot 7)$$

The other kinds of constituent quark spinors are transformed similarly as

$$u_-(-\mathbf{p}, s) \longrightarrow v_-(-\mathbf{p}, s) \quad \text{that is} \quad \mp v(\mathbf{p}, \mp) \longrightarrow \pm u(\mathbf{p}, \mp), \quad (\text{A}\cdot 8)$$

$$\bar{v}_+(\mathbf{p}, s) \longrightarrow \bar{u}_+(\mathbf{p}, s) \quad \text{that is} \quad \bar{v}(\mathbf{p}, \pm) \longrightarrow \bar{u}(\mathbf{p}, \pm), \quad (\text{A}\cdot 9)$$

$$\bar{v}_-(-\mathbf{p}, s) \longrightarrow \bar{u}_-(-\mathbf{p}, s) \quad \text{that is} \quad \pm \bar{u}(\mathbf{p}, \mp) \longrightarrow \mp \bar{v}(\mathbf{p}, \mp). \quad (\text{A}\cdot 10)$$

### A.3. Annihilation part of the composite meson spinor WF

As was explained in the text, depending upon the energy sign of the constituent spinor WF, there are four different types of (the annihilation part of) bi-spinor WF,  $U^{(+)}$ ,  $C^{(+)}$ ,  $D^{(+)}$  and  $V^{(+)}$ . Each type of the WF is decomposed into the irreducible components, which are constructed by using the usual spin composition.

The  $U^{(+)} \sim u_+\bar{v}_+(\sim u\bar{v})$  is decomposed into the pseudoscalar and vector components, which are constructed from the constituent spinors as

$$U_{P_s}^{(+)} = \frac{1}{i} \frac{1}{2m} \frac{1}{\sqrt{2}} (u_+(\mathbf{p}, +)\bar{v}_+(\mathbf{p}, -) - u_+(\mathbf{p}, -)\bar{v}_+(\mathbf{p}, +)) \quad (\text{A}\cdot 11)$$

$$= \frac{1}{i} \frac{1}{2m} \frac{1}{\sqrt{2}} (u(\mathbf{p}, +)\bar{v}(\mathbf{p}, -) - u(\mathbf{p}, -)\bar{v}(\mathbf{p}, +)) = \frac{1}{2\sqrt{2}} i\gamma_5(1 + iv \cdot \gamma)$$

$$\begin{aligned} U_{V_\mu}^{(+)} &= \frac{1}{2m} \begin{bmatrix} u_+(\mathbf{p}, +)\bar{v}_+(\mathbf{p}, +) \\ u_+(\mathbf{p}, -)\bar{v}_+(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}}(u_+(\mathbf{p}, +)\bar{v}_+(\mathbf{p}, -) + u_+(\mathbf{p}, -)\bar{v}_+(\mathbf{p}, +)) \end{bmatrix} \\ &= \frac{1}{2m} \begin{bmatrix} u(\mathbf{p}, +)\bar{v}(\mathbf{p}, +) \\ u(\mathbf{p}, -)\bar{v}(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}}(u(\mathbf{p}, +)\bar{v}(\mathbf{p}, -) + u(\mathbf{p}, -)\bar{v}(\mathbf{p}, +)) \end{bmatrix} \\ &= \frac{1}{4\sqrt{2}} (1 - iv \cdot \gamma) i\epsilon^{(\pm, 0)} \cdot \gamma (1 + iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i\epsilon^{(\pm, 0)} \cdot \gamma (1 + iv \cdot \gamma). \end{aligned} \quad (\text{A}\cdot 12)$$

The  $C^{(+)} \sim u_+\bar{v}_-(\sim u\bar{u})$  is decomposed into the scalar and axialvector components, which are constructed from the constituent spinors as

$$C_S^{(+)} = -\frac{1}{2m} \frac{1}{\sqrt{2}} (u_+(\mathbf{p}, +)\bar{v}_-(-\mathbf{p}, -) - u_+(\mathbf{p}, -)\bar{v}_-(-\mathbf{p}, +)) \quad (\text{A}\cdot 13)$$

$$= -\frac{1}{2m} \frac{1}{\sqrt{2}} (-u(\mathbf{p}, +)\bar{u}(\mathbf{p}, +) - u(\mathbf{p}, -)\bar{u}(\mathbf{p}, -)) = \frac{1}{2\sqrt{2}} (1 - iv \cdot \gamma)$$

$$C_{A_\mu}^{(+)} = -\frac{1}{2m} \begin{bmatrix} u_+(\mathbf{p}, +)\bar{v}_-(-\mathbf{p}, +) \\ u_+(\mathbf{p}, -)\bar{v}_-(-\mathbf{p}, -) \\ \frac{1}{\sqrt{2}}(u_+(\mathbf{p}, +)\bar{v}_-(-\mathbf{p}, -) + u_+(\mathbf{p}, -)\bar{v}_-(-\mathbf{p}, +)) \end{bmatrix}$$

$$\begin{aligned}
&= -\frac{1}{2m} \begin{bmatrix} u(\mathbf{p}, +)\bar{u}(\mathbf{p}, -) \\ -u(\mathbf{p}, -)\bar{u}(\mathbf{p}, +) \\ \frac{1}{\sqrt{2}}(-u(\mathbf{p}, +)\bar{u}(\mathbf{p}, +) + u(\mathbf{p}, -)\bar{u}(\mathbf{p}, -)) \end{bmatrix} \\
&= \frac{1}{4\sqrt{2}}(1 - iv \cdot \gamma) i\gamma_5 \epsilon^{(\pm, 0)} \cdot \gamma (1 - iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i\gamma_5 \epsilon^{(\pm, 0)} \cdot \gamma (1 - iv \cdot \gamma).
\end{aligned} \tag{A.14}$$

The  $D^{(+)} \sim u_- \bar{v}_+ (\sim v \bar{v})$  is decomposed into the scalar and axialvector components, which are constructed from the constituent spinors as

$$\begin{aligned}
D_S^{(+)} &= \frac{1}{2m} \frac{1}{\sqrt{2}} (u_-(-\mathbf{p}, +)\bar{v}_+(\mathbf{p}, -) - u_-(-\mathbf{p}, -)\bar{v}_+(\mathbf{p}, +)) \\
&= \frac{1}{2m} \frac{1}{\sqrt{2}} (-v(\mathbf{p}, -)\bar{v}(\mathbf{p}, -) - v(\mathbf{p}, +)\bar{v}(\mathbf{p}, +)) = \frac{1}{2\sqrt{2}} (1 + iv \cdot \gamma)
\end{aligned} \tag{A.15}$$

$$\begin{aligned}
D_{A_\mu}^{(+)} &= -\frac{1}{2m} \begin{bmatrix} u_-(-\mathbf{p}, +)\bar{v}_+(\mathbf{p}, +) \\ u_-(-\mathbf{p}, -)\bar{v}_+(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}}(u_-(-\mathbf{p}, +)\bar{v}_+(\mathbf{p}, -) + u_-(-\mathbf{p}, -)\bar{v}_+(\mathbf{p}, +)) \end{bmatrix} \\
&= -\frac{1}{2m} \begin{bmatrix} -v(\mathbf{p}, -)\bar{v}(\mathbf{p}, +) \\ v(\mathbf{p}, +)\bar{v}(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}}(-v(\mathbf{p}, -)\bar{v}(\mathbf{p}, -) + v(\mathbf{p}, +)\bar{v}(\mathbf{p}, +)) \end{bmatrix} \\
&= \frac{1}{4\sqrt{2}} (1 + iv \cdot \gamma) i\gamma_5 \epsilon^{(\pm, 0)} \cdot \gamma (1 + iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i\gamma_5 \epsilon^{(\pm, 0)} \cdot \gamma (1 + iv \cdot \gamma).
\end{aligned} \tag{A.16}$$

The  $V^{(+)} \sim u_- \bar{v}_- (\sim v \bar{u})$  is decomposed into the pseudoscalar and vector components, which are constructed from the constituent spinors as

$$\begin{aligned}
V_{P_s}^{(+)} &= i \frac{1}{2m} \frac{1}{\sqrt{2}} (u_-(-\mathbf{p}, +)\bar{v}_-(-\mathbf{p}, -) - u_-(-\mathbf{p}, -)\bar{v}_-(-\mathbf{p}, +)) \\
&= i \frac{1}{2m} \frac{1}{\sqrt{2}} (v(\mathbf{p}, -)\bar{u}(\mathbf{p}, +) - v(\mathbf{p}, +)\bar{u}(\mathbf{p}, -)) = \frac{1}{2\sqrt{2}} i\gamma_5 (1 - iv \cdot \gamma)
\end{aligned} \tag{A.17}$$

$$\begin{aligned}
V_{V_\mu}^{(+)} &= \frac{1}{2m} \begin{bmatrix} u_-(-\mathbf{p}, +)\bar{v}_-(-\mathbf{p}, +) \\ u_-(-\mathbf{p}, -)\bar{v}_-(-\mathbf{p}, -) \\ \frac{1}{\sqrt{2}}(u_-(-\mathbf{p}, +)\bar{v}_-(-\mathbf{p}, -) + u_-(-\mathbf{p}, -)\bar{v}_-(-\mathbf{p}, +)) \end{bmatrix} \\
&= \frac{1}{2m} \begin{bmatrix} -v(\mathbf{p}, -)\bar{u}(\mathbf{p}, -) \\ -v(\mathbf{p}, +)\bar{u}(\mathbf{p}, +) \\ \frac{1}{\sqrt{2}}(v(\mathbf{p}, -)\bar{u}(\mathbf{p}, +) + v(\mathbf{p}, +)\bar{u}(\mathbf{p}, -)) \end{bmatrix} \\
&= \frac{1}{4\sqrt{2}} (1 + iv \cdot \gamma) i\epsilon^{(\pm, 0)} \cdot \gamma (1 - iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i\epsilon^{(\pm, 0)} \cdot \gamma (1 - iv \cdot \gamma).
\end{aligned} \tag{A.18}$$

#### A.4. Creation part of the composite meson spinor $WF$

Creation part of composite meson spinor  $WF$  is obtained from the annihilation part, by applying the fundamental crossing rule Eqs.(A.7), (A.8), (A.9) and (A.10), as follows:

For  $U^{(-)} \sim v_+ \bar{u}_+ (\sim v \bar{u})$

$$U_{P_s}^{(-)} = \frac{1}{i} \frac{1}{2m} \frac{1}{\sqrt{2}} (v_+(\mathbf{p}, +)\bar{u}_+(\mathbf{p}, -) - v_+(\mathbf{p}, -)\bar{u}_+(\mathbf{p}, +)) \tag{A.19}$$

$$\begin{aligned}
&= \frac{1}{i} \frac{1}{2m} \frac{1}{\sqrt{2}} (v(\mathbf{p}, +) \bar{u}(\mathbf{p}, -) - v(\mathbf{p}, -) \bar{u}(\mathbf{p}, +)) = \frac{1}{2\sqrt{2}} i \gamma_5 (1 - iv \cdot \gamma) \\
U_{V_\mu^{(\pm,0)}}^{(-)} &= \frac{1}{2m} \begin{bmatrix} v_+(\mathbf{p}, +) \bar{u}_+(\mathbf{p}, +) \\ v_+(\mathbf{p}, -) \bar{u}_+(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}} (v_+(\mathbf{p}, +) \bar{u}_+(\mathbf{p}, -) + v_+(\mathbf{p}, -) \bar{u}_+(\mathbf{p}, +)) \end{bmatrix} \\
&= \frac{1}{2m} \begin{bmatrix} v(\mathbf{p}, +) \bar{u}(\mathbf{p}, +) \\ v(\mathbf{p}, -) \bar{u}(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}} (v(\mathbf{p}, +) \bar{u}(\mathbf{p}, -) + v(\mathbf{p}, -) \bar{u}(\mathbf{p}, +)) \end{bmatrix} \quad (\text{A}\cdot 20) \\
&= \frac{1}{4\sqrt{2}} (1 + iv \cdot \gamma) i \tilde{\epsilon}^{(\pm,0)} \cdot \gamma (1 - iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i \tilde{\epsilon}^{(\pm,0)} \cdot \gamma (1 - iv \cdot \gamma).
\end{aligned}$$

For  $C^{(-)} \sim v_+ \bar{u}_- (\sim v \bar{v})$

$$\begin{aligned}
C_S^{(-)} &= -\frac{1}{2m} \frac{1}{\sqrt{2}} (v_+(\mathbf{p}, +) \bar{u}_-(-\mathbf{p}, -) - v_+(\mathbf{p}, -) \bar{u}_-(-\mathbf{p}, +)) \quad (\text{A}\cdot 21) \\
&= -\frac{1}{2m} \frac{1}{\sqrt{2}} (v(\mathbf{p}, +) \bar{v}(\mathbf{p}, +) + v(\mathbf{p}, -) \bar{v}(\mathbf{p}, -)) = \frac{1}{2\sqrt{2}} (1 + iv \cdot \gamma)
\end{aligned}$$

$$\begin{aligned}
C_{A_\mu^{(\pm,0)}}^{(-)} &= -\frac{1}{2m} \begin{bmatrix} v_+(\mathbf{p}, +) \bar{u}_-(-\mathbf{p}, +) \\ v_+(\mathbf{p}, -) \bar{u}_-(-\mathbf{p}, -) \\ \frac{1}{\sqrt{2}} (v_+(\mathbf{p}, +) \bar{u}_-(-\mathbf{p}, -) + v_+(\mathbf{p}, -) \bar{u}_-(-\mathbf{p}, +)) \end{bmatrix} \\
&= -\frac{1}{2m} \begin{bmatrix} -v(\mathbf{p}, +) \bar{v}(\mathbf{p}, -) \\ v(\mathbf{p}, -) \bar{v}(\mathbf{p}, +) \\ \frac{1}{\sqrt{2}} (v(\mathbf{p}, +) \bar{v}(\mathbf{p}, +) - v(\mathbf{p}, -) \bar{v}(\mathbf{p}, -)) \end{bmatrix} \quad (\text{A}\cdot 22) \\
&= \frac{1}{4\sqrt{2}} (1 + iv \cdot \gamma) i \gamma_5 \tilde{\epsilon}^{(\pm,0)} \cdot \gamma (1 + iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i \gamma_5 \tilde{\epsilon}^{(\pm,0)} \cdot \gamma (1 + iv \cdot \gamma).
\end{aligned}$$

For  $D^{(-)} \sim v_- \bar{u}_+ (\sim u \bar{u})$

$$\begin{aligned}
D_S^{(-)} &= \frac{1}{2m} \frac{1}{\sqrt{2}} (v_-(-\mathbf{p}, +) \bar{u}_+(\mathbf{p}, -) - v_-(-\mathbf{p}, -) \bar{u}_+(\mathbf{p}, +)) \quad (\text{A}\cdot 23) \\
&= \frac{1}{2m} \frac{1}{\sqrt{2}} (u(\mathbf{p}, -) \bar{u}(\mathbf{p}, -) + u(\mathbf{p}, +) \bar{u}(\mathbf{p}, +)) = \frac{1}{2\sqrt{2}} (1 - iv \cdot \gamma)
\end{aligned}$$

$$\begin{aligned}
D_{A_\mu^{(\pm,0)}}^{(-)} &= -\frac{1}{2m} \begin{bmatrix} v_-(-\mathbf{p}, +) \bar{u}_+(\mathbf{p}, +) \\ v_-(-\mathbf{p}, -) \bar{u}_+(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}} (v_-(-\mathbf{p}, +) \bar{u}_+(\mathbf{p}, -) + v_-(-\mathbf{p}, -) \bar{u}_+(\mathbf{p}, +)) \end{bmatrix} \\
&= -\frac{1}{2m} \begin{bmatrix} u(\mathbf{p}, -) \bar{u}(\mathbf{p}, +) \\ -u(\mathbf{p}, +) \bar{u}(\mathbf{p}, -) \\ \frac{1}{\sqrt{2}} (u(\mathbf{p}, -) \bar{u}(\mathbf{p}, -) - u(\mathbf{p}, +) \bar{u}(\mathbf{p}, +)) \end{bmatrix} \quad (\text{A}\cdot 24) \\
&= \frac{1}{4\sqrt{2}} (1 - iv \cdot \gamma) i \gamma_5 \tilde{\epsilon}^{(\pm,0)} \cdot \gamma (1 - iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i \gamma_5 \tilde{\epsilon}^{(\pm,0)} \cdot \gamma (1 - iv \cdot \gamma).
\end{aligned}$$

For  $V^{(-)} \sim v_- \bar{u}_- (\sim u \bar{v})$

$$V_{P_s}^{(-)} = i \frac{1}{2m} \frac{1}{\sqrt{2}} (v_-(-\mathbf{p}, +) \bar{u}_-(-\mathbf{p}, -) - v_-(-\mathbf{p}, -) \bar{u}_-(-\mathbf{p}, +)) \quad (\text{A}\cdot 25)$$

$$\begin{aligned}
&= i \frac{1}{2m} \frac{1}{\sqrt{2}} (u(\mathbf{p}, -) \bar{v}(\mathbf{p}, +) - u(\mathbf{p}, +) \bar{v}(\mathbf{p}, -)) = \frac{1}{2\sqrt{2}} i \gamma_5 (1 + iv \cdot \gamma) \\
V_{\mu}^{(-)} &= \frac{1}{2m} \begin{bmatrix} v_-(-\mathbf{p}, +) \bar{u}_-(-\mathbf{p}, +) \\ v_-(-\mathbf{p}, -) \bar{u}_-(-\mathbf{p}, -) \\ \frac{1}{\sqrt{2}} (v_-(-\mathbf{p}, +) \bar{u}_-(-\mathbf{p}, -) + v_-(-\mathbf{p}, -) \bar{u}_-(-\mathbf{p}, +)) \end{bmatrix} \\
&= \frac{1}{2m} \begin{bmatrix} -u(\mathbf{p}, -) \bar{v}(\mathbf{p}, -) \\ -u(\mathbf{p}, +) \bar{v}(\mathbf{p}, +) \\ \frac{1}{\sqrt{2}} (u(\mathbf{p}, -) \bar{v}(\mathbf{p}, +) + u(\mathbf{p}, +) \bar{v}(\mathbf{p}, -)) \end{bmatrix} \quad (\text{A}\cdot 26) \\
&= \frac{1}{4\sqrt{2}} (1 - iv \cdot \gamma) i \tilde{\epsilon}^{(\pm, 0)} \cdot \gamma (1 + iv \cdot \gamma) = \frac{1}{2\sqrt{2}} i \tilde{\epsilon}^{(\pm, 0)} \cdot \gamma (1 + iv \cdot \gamma).
\end{aligned}$$

#### A.5. Representation by local meson field

We can simply combine the annihilation and creation parts of the composite spinor WF into the local meson field (neglecting the freedom of internal space-time  $x$ ).

For non-relativistic type spinors

$$\frac{1}{2\sqrt{2}} i \gamma_5 (1 + \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) P_s^{(NR)}(X), \quad \frac{1}{2\sqrt{2}} i \gamma_\mu (1 + \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) V_\mu^{(NR)}(X), \quad (\text{A}\cdot 27)$$

where  $P_s^{(NR)}(X)$  and  $V_\mu^{(NR)}(X)$  represents the local pseudoscalar and vector meson field operators of non-relativistic-type, respectively.

For relativistic  $\bar{q}$ -type spinors

$$\frac{1}{2\sqrt{2}} (1 - \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) S^{\bar{q}}(X), \quad \frac{1}{2\sqrt{2}} i \gamma_5 \gamma_\mu (1 - \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) A_\mu^{\bar{q}}(X), \quad (\text{A}\cdot 28)$$

where  $S^{\bar{q}}(X)$  and  $A_\mu^{\bar{q}}(X)$  represents the local pseudoscalar and vector meson field operators of  $\bar{q}$ -type, respectively.

For relativistic  $q$ -type spinors

$$\frac{1}{2\sqrt{2}} (1 + \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) S^q(X), \quad \frac{1}{2\sqrt{2}} i \gamma_5 \gamma_\mu (1 + \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) A_\mu^q(X), \quad (\text{A}\cdot 29)$$

where  $S^q(X)$  and  $A_\mu^q(X)$  represents the local pseudoscalar and vector meson field operators of  $D$ -type, respectively.

For extremely relativistic-type spinors

$$\frac{1}{2\sqrt{2}} i \gamma_5 (1 - \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) P_s^{(ER)}(X), \quad \frac{1}{2\sqrt{2}} i \gamma_\mu (1 - \gamma_\nu \frac{\partial_\nu}{\sqrt{\partial^2}}) V_\mu^{(ER)}(X), \quad (\text{A}\cdot 30)$$

where  $P_s^{(ER)}(X)$  and  $V_\mu^{(ER)}(X)$  represents the local pseudoscalar and vector meson field operators of extremely relativistic-type, respectively.

These local meson field representation of the composite quark spinor WF guarantees the crossing symmetry of the interaction among composite mesons, which are induced from the crossing symmetric interaction between constituent quarks and antiquarks.



## Appendix B

— orthonormality relation for spinor WF —

### B.1. Bargmann-Wigner (BW) bases

The Pauli-conjugate of the annihilation parts  $W_i^{(+)}$  of WF,  $\bar{W}_i^{(-)} (\equiv \gamma_4 W_i^{(+)\dagger} \gamma_4)$ , are related with the creation parts as

$$\begin{aligned} (\bar{W}_i^{(-)}) &= (\bar{U}_{P_s}^{(-)}, \bar{U}_{V_\mu}^{(-)}, \bar{C}_S^{(-)}, \bar{C}_{A_\mu}^{(-)}, \bar{D}_S^{(-)}, \bar{D}_{A_\mu}^{(-)}, \bar{V}_{P_s}^{(-)}, \bar{V}_{V_\mu}^{(-)}) \\ &= (U_{P_s}^{(-)}, U_{V_\mu}^{(-)}, D_S^{(-)}, D_{A_\mu}^{(-)}, C_S^{(-)}, C_{A_\mu}^{(-)}, V_{P_s}^{(-)}, V_{V_\mu}^{(-)}). \end{aligned} \quad (\text{B.1})$$

They are orthonormal to the annihilation WF  $W_i^{(+)}$  as

$$\begin{aligned} \langle \bar{W}_i^{(-)} W_j^{(+)} \rangle &= \epsilon_i \delta_{ij} \\ (\epsilon_{P_s^{(NR)}}, \epsilon_{V_\mu^{(NR)}}, \epsilon_{S^{(\bar{q})}}, \epsilon_{A_\mu^{(\bar{q})}}, \epsilon_{S^{(q)}}, \epsilon_{A_\mu^{(q)}}, \epsilon_{P_s^{(ER)}}, \epsilon_{V_\mu^{(ER)}}) &= (-1, -1, 1, 1, 1, 1, -1, -1). \end{aligned} \quad (\text{B.2})$$

By using this orthonormality relation we can decompose the general spinor WF  $\Psi^{(+)}$  into the meson components,  $(\phi_i^{(+)}) = (P_s^{(+)(NR,ER)}, S^{(+)(\bar{q},q)}, V_\mu^{(+)(NR,ER)}, A_\mu^{(+)(\bar{q},q)})$ .

$$\Psi^{(+)} = \sum_i W_i^{(+)} \phi_i^{(+)}. \quad \phi_i^{(+)} = \epsilon_i \langle \bar{W}_i^{(-)} \Psi^{(+)} \rangle \quad (\text{B.3})$$

### B.2. chiral bases—light quark $q\bar{q}$ -meson systems

For description of the light quark  $q\bar{q}$ -meson systems the chiral  $(N, E)$  bases of spinor WF are expected to be more effective. They are obtained by the linear combination of BW bases as explained in the text.

The annihilation WF  $W_i^{(+)}$  and creation WF  $W_i^{(-)}$  are given, respectively, by

$$\begin{aligned} (W_i^{(+)}(P)) &= (U_{P_s}^{(+)(N)}, C_S^{(+)(N)}, U_{V_\mu}^{(+)(N)}, C_{A_\mu}^{(+)(N)}; U_{P_s}^{(+)(E)}, C_S^{(+)(N)}, U_{V_\mu}^{(+)(E)}, C_{A_\mu}^{(+)(E)}) \\ &= \frac{1}{2}(i\gamma_5, 1, i\tilde{\gamma}_\mu, i\gamma_5\tilde{\gamma}_\mu; -\gamma_5 v \cdot \gamma, -v \cdot \gamma, -i\sigma_{\mu\nu}v_\nu, \gamma_5\sigma_{\mu\nu}v_\nu) \\ (W_i^{(-)}(P)) &= (U_{P_s}^{(-)(N)}, C_S^{(-)(N)}, U_{V_\mu}^{(-)(N)}, C_{A_\mu}^{(-)(N)}; U_{P_s}^{(-)(E)}, C_S^{(-)(N)}, U_{V_\mu}^{(-)(E)}, C_{A_\mu}^{(-)(E)}) \\ &= \frac{1}{2}(i\gamma_5, 1, i\tilde{\gamma}_\mu, i\gamma_5\tilde{\gamma}_\mu; \gamma_5 v \cdot \gamma, v \cdot \gamma, i\sigma_{\mu\nu}v_\nu, -\gamma_5\sigma_{\mu\nu}v_\nu), \end{aligned} \quad (\text{B.4})$$

which are related through crossing rules of constituent spinors with each other.

The Pauli-conjugate of the creation spinor  $\bar{W}_i^{(-)} (\equiv \gamma_4 W_i^{(+)\dagger} \gamma_4)$  are equal to the annihilation  $W_i^{(-)}$ .

$$\begin{aligned} \bar{W}_i^{(-)}(P) (\equiv \gamma_4 W_i^{(+)\dagger} \gamma_4) &= W_i^{(-)}(P) \\ \text{for } \phi_i &= P_s^{(N)}, S^{(N)}, V_\mu^{(N)}, A_\mu^{(N)}; P_s^{(E)}, S^{(N)}, V_\mu^{(E)}, A_\mu^{(E)}. \end{aligned} \quad (\text{B.5})$$

They satisfy orthonormality relation:

$$\langle \bar{W}_i^{(-)} W_j^{(+)} \rangle = \epsilon_i \delta_{ij}$$

$$\begin{aligned}
(\epsilon_i) &= (\epsilon_{P_s^{(N)}}, \epsilon_{S^{(N)}}, \epsilon_{V_\mu^{(N)}}, \epsilon_{A_\mu^{(N)}}; \epsilon_{P_s^{(E)}}, \epsilon_{S^{(N)}}, \epsilon_{V_\mu^{(E)}}, \epsilon_{A_\mu^{(E)}}) \\
&= (-1, 1, -1, 1; 1, -1, -1, 1)
\end{aligned} \tag{B.6}$$

By using this orthonormality relation the general WF  $\Psi^{(+)}$  are decomposed into the meson components  $(\phi_i^{(+)}) = (P_s^{(+)(N,E)}, S^{(+)(N,E)}, V_\mu^{(+)(N,E)}, A_\mu^{(+)(N,E)})$ .

$$\Psi^{(+)} = \sum_i W_i^{(+)} \phi_i^{(+)}. \quad \phi_i^{(+)} = \epsilon_i \langle \bar{W}_i^{(-)} \Psi^{(+)} \rangle \tag{B.7}$$

### Appendix C

—— Chiral transformation for spinor WF of light quark  $q\bar{q}$  mesons ——

$SU(3)$  chiral transformation for the annihilation part of spinor WF of light quark  $q\bar{q}$  mesons is given by

$$\Psi_A^{(+)B}(P, x) \rightarrow [e^{i\frac{\alpha^i \lambda^i}{2} \gamma_5} \Psi^{(+)}(P, x) e^{i\frac{\alpha^i \lambda^i}{2} \gamma_5}]_A^B. \tag{C.1}$$

For the infinitesimal transformation  $\{i\frac{\alpha^i \lambda^i}{2} \gamma_5, \Psi^{(+)}(P, x)\}$  the respective meson spinor WF are transformed as

$$\begin{aligned}
P_s^{(N)} \cdot U_{P_s}^{(N)} &\rightarrow \left\{ \frac{\alpha^i \lambda^i}{2}, P_s^{(N)} \right\} \cdot \frac{1}{2} \{i\gamma_5, U_{P_s}^{(N)}\} = d^{ijk} \alpha^i P_s^{(N)j} \frac{\lambda^k}{2} \cdot C_S^{(N)} \\
S^{(N)} \cdot C_S^{(N)} &\rightarrow \left\{ \frac{\alpha^i \lambda^i}{2}, S^{(N)} \right\} \cdot \frac{1}{2} \{i\gamma_5, C_S^{(N)}\} = d^{ijk} \alpha^i S^{(N)j} \frac{\lambda^k}{2} \cdot U_{P_s}^{(N)} \\
V_\mu^{(N)} \cdot U_{V_\mu}^{(N)} &\rightarrow \left[ \frac{\alpha^i \lambda^i}{2}, V_\mu^{(N)} \right] \cdot \frac{1}{2} [i\gamma_5, U_{V_\mu}^{(N)}] = -f^{ijk} \alpha^i V_\mu^{(N)j} \frac{\lambda^k}{2} \cdot C_{A_\mu}^{(N)} \\
A_\mu^{(N)} \cdot C_{A_\mu}^{(N)} &\rightarrow \left[ \frac{\alpha^i \lambda^i}{2}, A_\mu^{(N)} \right] \cdot \frac{1}{2} [i\gamma_5, C_{A_\mu}^{(N)}] = -f^{ijk} \alpha^i A_\mu^{(N)j} \frac{\lambda^k}{2} \cdot U_{V_\mu}^{(N)} \\
P_s^{(E)} \cdot U_{P_s}^{(E)} &\rightarrow \left[ \frac{\alpha^i \lambda^i}{2}, P_s^{(E)} \right] \cdot \frac{1}{2} [i\gamma_5, U_{P_s}^{(E)}] = -f^{ijk} \alpha^i P_s^{(E)j} \frac{\lambda^k}{2} \cdot C_S^{(E)} \\
S^{(E)} \cdot C_S^{(E)} &\rightarrow \left[ \frac{\alpha^i \lambda^i}{2}, S^{(E)} \right] \cdot \frac{1}{2} [i\gamma_5, C_S^{(E)}] = -f^{ijk} \alpha^i S^{(E)j} \frac{\lambda^k}{2} \cdot U_{P_s}^{(E)} \\
V_\mu^{(E)} \cdot U_{V_\mu}^{(E)} &\rightarrow \left\{ \frac{\alpha^i \lambda^i}{2}, V_\mu^{(E)} \right\} \cdot \frac{1}{2} \{i\gamma_5, U_{V_\mu}^{(E)}\} = d^{ijk} \alpha^i V_\mu^{(E)j} \frac{\lambda^k}{2} \cdot C_{A_\mu}^{(E)} \\
A_\mu^{(E)} \cdot C_{A_\mu}^{(E)} &\rightarrow \left\{ \frac{\alpha^i \lambda^i}{2}, A_\mu^{(E)} \right\} \cdot \frac{1}{2} \{i\gamma_5, C_{A_\mu}^{(E)}\} = d^{ijk} \alpha^i A_\mu^{(E)j} \frac{\lambda^k}{2} \cdot U_{V_\mu}^{(E)}. \tag{C.2}
\end{aligned}$$

For the finite  $U(1)$  chiral transformation

$$\Psi_A^{(+)B}(P, x) \rightarrow [e^{i\alpha \gamma_5} \Psi^{(+)}(P, x) e^{i\alpha \gamma_5}]_A^B, \tag{C.3}$$

the spinor WF for  $V_\mu^{(N)}$ ,  $A_\mu^{(N)}$ ,  $P_s^{(E)}$  and  $S^{(E)}$  are invariant, while

$$\begin{aligned}
P_s^{(N)} \cdot U_{P_s}^{(N)} &\rightarrow P_s^{(N)} \cdot (U_{P_s}^{(N)} \cos 2\alpha - C_s^{(N)} \sin 2\alpha), \\
S^{(N)} \cdot C_S^{(N)} &\rightarrow S^{(N)} \cdot (C_s^{(N)} \cos 2\alpha + U_{P_s}^{(N)} \sin 2\alpha), \\
V_\mu^{(E)} \cdot U_{V_\mu}^{(E)} &\rightarrow V_\mu^{(E)} \cdot (U_{V_\mu}^{(E)} \cos 2\alpha + C_{A_\mu}^{(E)} \sin 2\alpha), \\
A_\mu^{(E)} \cdot C_{A_\mu}^{(E)} &\rightarrow A_\mu^{(E)} \cdot (C_{A_\mu}^{(E)} \cos 2\alpha - U_{V_\mu}^{(E)} \sin 2\alpha). \tag{C.4}
\end{aligned}$$

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